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### Some calculations in a three-sector model

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W.M. van den Goorbergh

Some calculations in a three-sector model



Research memorandum

TILBURG UNIVERSITY  
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SOME CALCULATIONS IN A THREE-SECTOR MODEL

by

W.M. van den Goorbergh

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## §1 Introduction

This paper has been written as an appendix to a paper which I have presented at the annual meeting of "De Vereniging voor de Staathuishoudkunde" in December 1975, titled 'De structurele vraag naar arbeid in een drie-sectorenmodel'\*) . It is however not strictly necessary to study that paper first before reading this one, because the model which is presented in the main paper, is treated here too. The main emphasis is however on rather technical problems, especially on problems of stability. So for a more detailed treatment of the properties of the model and of its possibilities of application to actual economic development, I refer to the main paper.

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\*) The English version of that paper will be published in the quarterly review 'De Economist', Volume 124 (1976), issue 3.

## §2 The static equilibrium model

Under technological conditions with fixed coefficients the services of a single grade of labour and of a single type of capital good (machines) are the inputs for producing three types of commodities; capital goods ( $i$ ), industrial consumer goods ( $c_1$ ) and consumer goods as services ( $c_2$ ). Labour and machines are freely transferable from one sector to another. Input-prices are the same in all industries and commodity-prices are equal to the sum of labourcosts and gross profits (i.e. depreciation of capital included), each per product unit. Wages are all spent on consumer goods in a constant ratio and profits are all spent on machines. The supply of labour ( $\tilde{l}_a$ ) and the stock of capital goods ( $\tilde{k}$ ), which is supposed to be fully employed<sup>1)</sup>, are exogeneously determined. The price of good  $c_1$  is set to one. The model:

$$(2.1) \quad \alpha_1 \tilde{i} + \alpha_1 \tilde{c}_1 + \alpha_2 \tilde{c}_2 = \tilde{l}_v \quad : \text{demand function for labour}$$

$$(2.2) \quad \kappa_1 \tilde{i} + \kappa_1 \tilde{c}_1 + \kappa_2 \tilde{c}_2 = \tilde{k} \quad : \text{production possibility function of machines}$$

$$(2.3) \quad \tilde{p}_1 = \alpha_1 \tilde{p}_L + \kappa_1 \tilde{p}_i \tilde{r}$$

$$(2.4) \quad \tilde{p}_2 = \alpha_2 \tilde{p}_L + \kappa_2 \tilde{p}_i \tilde{r}$$

$$(2.5) \quad \tilde{p}_i = \alpha_i \tilde{p}_L + \kappa_i \tilde{p}_i \tilde{r}$$

} price functions

$$(2.6) \quad \frac{\tilde{c}_1}{\tilde{c}_2} = \frac{\gamma}{1-\gamma} \cdot \frac{\tilde{p}_2}{\tilde{p}_1} \quad : \text{spendings out of wages}$$

$$(2.7) \quad \tilde{\gamma} = \tilde{k} \cdot \tilde{r} \quad : \text{spendings out of profits}$$

<sup>1)</sup> In appendix A the hypothesis of full utilisation of capacity is dropped and some calculations are made based on a model with exogeneously determined investment.

$$(2.8) \quad \tilde{p}_1 = 1 \quad : \text{numeraire}$$

The meaning of the symbols:

$\alpha_j$	$j = i, 1, 2$	: labour-output ratio in sector $j$
$\kappa_j$	$j = i, 1, 2$	: capital-output ratio in sector $j$
$\tilde{z}_i$		: production volume good $i$
$\tilde{c}_1$		: production volume good $c_1$
$\tilde{c}_2$		: production volume good $c_2$
$\tilde{\ell}_v$		: demand for labour
$\tilde{k}$		: stock of capital goods
$\tilde{p}_j$	$j = i, 1, 2$	: commodity-price good $j$
$\tilde{p}_L$		: nominal wage rate
$\tilde{r}$		: profit-rate (depreciation included)
$\gamma$		: parameter of consumer preference

The model, featuring eight equations and nine unknowns, can be completed in two ways; first by assuming full employment of labour

$$(2.9)^a \quad \tilde{\ell}_v = \tilde{\ell}_a$$

and second by fixing the distribution of income

$$(2.9)^b \quad \tilde{p}_L = \tilde{p}_L$$

In the first case an income distribution is established supporting full employment of labour, and in the second case high wages cause unemployment and low wages overemployment. For reasons of simplicity  $\kappa_2$  is set equal to zero. This has no consequences for the qualitative properties of the model. Equations (2.10), the well known factor price frontier, and (2.11), the aggregated demand function for labour, are easily



deduced from the model equations:

$$(2.10) \quad \tilde{w} \equiv \frac{\tilde{p}_L}{\tilde{p}_1} = \frac{1 - \kappa_i \tilde{r}}{\alpha_1 + D \tilde{r}} \quad D = \kappa_1 \alpha_i - \kappa_i \alpha_1$$

$$(2.11) \quad \tilde{\ell}_v = \frac{1}{\gamma} \cdot \frac{1}{\kappa_1} \cdot (\alpha_1 + D \tilde{r}) \cdot \tilde{k}$$

Assuming the  $c_1$ -goods sector more capital-intensive than the investment-goods sector, which means  $D > 0$ , the relation between income distribution and demand for labour is represented in figure 1.

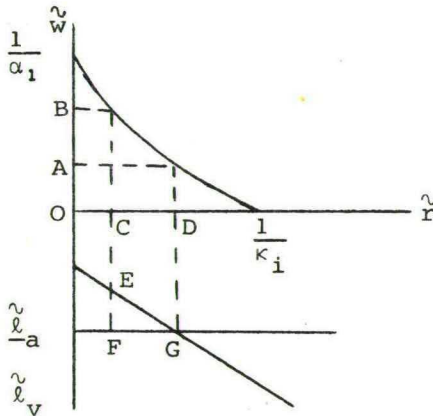


figure 1: the relation between demand for labour and income distribution.

Full employment (DG) is realised at the intersection G of the demand function for labour (2.11) and the inelastic supply function of labour ( $\tilde{\ell}_a$ ) at a profit-rate OD. According to the factor price frontier the real wage-rate is OA. A higher real wage-rate, say OB, makes the profit-rate lower (OC) and causes unemployment (EF).

These results are summarized in two examples in table 1.

Data:  $\alpha_i = \frac{5}{6}$      $\alpha_1 = \frac{1}{2}$      $\alpha_2 = 1$      $\frac{\tilde{l}}{\tilde{a}} = 40$      $\gamma = \frac{1}{2}$

$\kappa_i = \frac{5}{3}$      $\kappa_1 = 5$      $\kappa_2 = 0$      $\frac{\tilde{k}}{\tilde{a}} = 120$      $D = \frac{10}{3}$

Solution:

Table 1 Two examples of the static equilibrium model

		a) $\frac{\tilde{l}_v}{\tilde{a}} = \frac{\tilde{l}_a}{\tilde{a}}$	b) $\tilde{p}_L = 1,375$
wages	$\tilde{l} \cdot \tilde{p}_L$	$40 \times 1 = 40$	$32 \times 1,375 = 44$
profits	$\tilde{k} \cdot \tilde{p}_i \cdot r$	$120 \times 1 \times 0,1 = \frac{12}{52}$	$120 \times 1,25 \times 0,05 = \frac{7,5}{51,5}$
$c_1$ -consumption	$\tilde{c}_1 \cdot \tilde{p}_1$	$20 \times 1 = 20$	$22 \times 1 = 22$
$c_2$ -consumption	$\tilde{c}_2 \cdot \tilde{p}_2$	$20 \times 1 = 20$	$16 \times 1,375 = 22$
investment	$\tilde{i} \cdot \tilde{p}_i$	$12 \times 1 = \frac{12}{52}$	$6 \times 1,25 = \frac{7,5}{51,5}$
employment $c_1$ -sector	$\alpha_1 \cdot \tilde{c}_1$	$\frac{1}{2} \times 20 = 10$	$\frac{1}{2} \times 22 = 11$
employment $c_2$ -sector	$\alpha_2 \cdot \tilde{c}_2$	$1 \times 20 = 20$	$1 \times 16 = 16$
employment i-sector	$\alpha_i \cdot \tilde{i}$	$\frac{5}{6} \times 12 = \frac{10}{40}$	$\frac{5}{6} \times 6 = 5$
total employment		40	32

### §3. The process of short term adjustment

It follows from the preceding paragraph that full employment can be realised for only one specific distribution of income. Are there however on short term, i.e. within the period that the capital stock is supposed to be given and constant, forces tending to full employment, if it is not realised as a result of a 'wrong' distribution of income? The answer might be found in a 'Phillips-curve' relation between the rate of rise in wages and the (rising) proportion of unemployment.

Now two versions are presented concerning the process of short term adjustment:

- a) The labour market is supposed to operate in a strong way, i.e. that the real wage-rate is changing until full employment of labour is realised.
- b) The labour market is supposed to operate in a weak way, i.e. that the real wage-rate won't change anymore, unless demand for labour changes.

It will be proved that in case a) full employment is the result of the operation of the labour market, but not always in a stable way, and that in case b) unemployment or total instability are caused by a wage-rate that is initially too high.

The processes of adjustment are formulated in terms of relative differences from the equilibrium solution of the model. So  $\hat{x}$  being the real and  $\hat{x}_0$  being the equilibrium value of a variable, the relative difference  $x$  of that variable is defined as

$$(3.1) \quad x \equiv \frac{\hat{x} - \hat{x}_0}{\hat{x}_0} \quad ^2)$$

In the equilibrium neighbourhood the first difference of this new defined variable ( $\Delta x$ )

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<sup>2)</sup> Mutations in the profit-rate are usually measured in points of percentages so  $r \equiv \hat{r} - \hat{r}_0$ .

$$(3.2) \quad \Delta x \equiv x - x_{-1}$$

is approximately equal to the relative mutation of the variable with respect to the preceding period.

Now the factor price frontier (2.10) and the aggregated demand function for labour (2.11) can be reformulated as follows:

$$(3.3) \quad w = -Ar \quad ^3) \quad \text{with} \quad A = \frac{\kappa_1 \alpha_i}{(\alpha_1 + Dr_0)(1 - \kappa_i \tilde{r}_0)} > 0$$

$$(3.4) \quad \ell = Br \quad ^4) \quad \text{with} \quad B = \frac{D}{\alpha_1 + Dr_0} > 0$$

If there is a certain lag in reaction of changes of the wage-rate to labour market conditions and if an initial disturbance of the equilibrium wage-rate is denoted by  $\Delta p_L$ <sup>5)</sup>, the two versions of labour market reactions can be formulated as follows:

$$(3.5)^a \quad \Delta w = \beta_1 \ell_{-1} + \Delta p_L \quad ^6) \quad (\text{strong version})$$

$$(3.5)^b \quad \Delta w = \beta_2 \Delta \ell_{-1} + \Delta p_L \quad ^6) \quad (\text{weak version})$$

The model consisting of the equations (3.3), (3.4) and (3.5)<sup>a</sup>, can be reduced to a set of difference equations:

<sup>3)</sup> Second order effects are neglected. The exact solution is:  
 $w = -Ar - Bwr$ .

<sup>4)</sup> On short term there are no changes in the capital stock.

<sup>5)</sup> An incidental wage-push is defined by:

$$\Delta p_{L_t} = 1 \text{ for } t = 1 \\ = 0 \text{ for } t \neq 1$$

According to definition (3.2) this is identical to:

$$p_{L_t} = 1 \text{ for all } t \geq 1$$

A permanent wage-push is defined by:  $\Delta p_{L_t} = 1$  for all  $t \geq 1$

<sup>6)</sup> The coefficients of reaction in the labour market are denoted by  $\beta_1$  resp.  $\beta_2$ .



$$(3.6) \quad \begin{Bmatrix} w \\ r \\ l \end{Bmatrix}_t = (1 - \beta_1 \frac{B}{A}) \begin{Bmatrix} w \\ r \\ l \end{Bmatrix}_{t-1} + \begin{Bmatrix} 1 \\ -\frac{1}{A} \\ -\frac{B}{A} \end{Bmatrix} \Delta p_{Lt}$$

For an incidental wage-push the structural or trend solution of the process of adjustment is equal to the equilibrium solution of full employment. This follows from the solution of the set of difference equations (3.6), which is given for an incidental 1% wage-push in period 1 by:

$$(3.7) \quad \begin{Bmatrix} w \\ r \\ l \end{Bmatrix}_t = \left\{ 1 - \beta_1 \frac{B}{A} \right\}^{t-1} \cdot \begin{Bmatrix} 1 \\ -\frac{1}{A} \\ -\frac{B}{A} \end{Bmatrix}$$

The stability and the character of the adjustment process depend on the quantitative values  $A$ ,  $B$  and  $\beta_1$ . The process is

- a. asymptotically stable if  $0 < \beta_1 < \frac{B}{A}$
- b. quasi stationary if  $\beta_1 = \frac{B}{A}$
- c. oscillatory stable if  $\frac{A}{B} < \beta_1 < 2 \frac{A}{B}$
- d. oscillating with constant amplitude if  $\beta_1 = 2 \frac{A}{B}$
- e. unstable with explosive oscillations if  $\beta_1 > 2 \frac{A}{B}$

In a diagram (fig. 2) the oscillatory stable adjustment process (case c) is illustrated. The factor price frontier (2.10) and the aggregated demand function for labour (2.11) are drawn in the first and second quarter of the diagram (cfr. fig. 1, page 4). The (strong) reactions in the labour market are denoted by curve (a)-(a) in the third quarter<sup>7)</sup>, and a 45°-line

<sup>7)</sup> See for the exact algebraic formulation of curve (a)-(a) and an alternative graphic representation of the adjustment process appendix B.

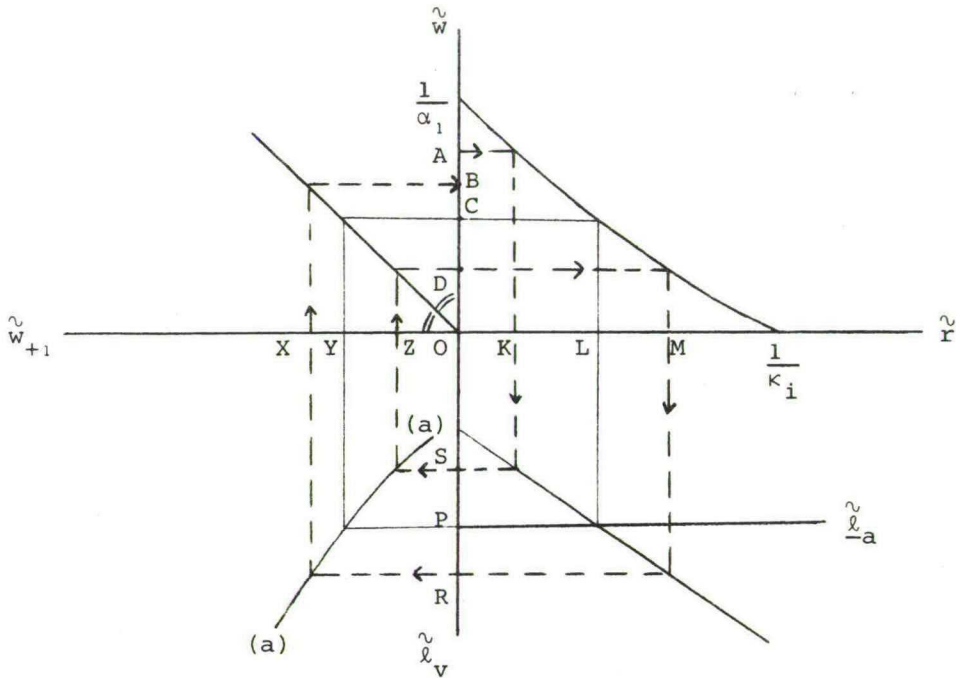


Fig. 2: the process of short term adjustment (the strong case)

in the fourth quarter is transforming the wage-rate, measured on the Western horizontal axis into the wage-rate, measured on the Northern vertical axis.

Now from the diagram it is easy to see, that a wage-rate set higher than the equilibrium-rate ( $OA > OC$ ), at first causes a low profit rate ( $OK$ ) and unemployment ( $PS$ ). Reactions in the labour-market reduce the wage-rate to  $OZ (< OC)$  under the equilibrium-rate, so overemployment ( $PR$ ) results and wages are pushed up again ( $OX > OZ$ , but  $OC < OX < OA$ ). A new round of wage adjustment begins but in a more close neighbourhood of equilibrium. The adjustment process will stop if full employment ( $OP$ ) is reached;  $OC$  and  $OL$  denote the resulting equilibrium rates of wages and profits.

The model consisting of the equations (3.3), (3.4) and (3.5)<sup>b</sup>, can be reduced to a set of difference equations:

$$(3.8) \quad \begin{Bmatrix} w \\ r \\ \ell \end{Bmatrix}_t = (-\beta_2 \frac{B}{A}) \begin{Bmatrix} w \\ r \\ \ell \end{Bmatrix}_{t-1} + \begin{Bmatrix} 1 \\ -\frac{1}{A} \\ -\frac{B}{A} \end{Bmatrix} P_{Lt}.$$

For a (positive) incidental wage-push the structural or trend solution of the process of adjustment will be unequal to the equilibrium solution in such a way that there will be unemployment, a wage-rate higher than, and a profit-rate lower than equilibrium. This follows from the solution of the set of difference equations (3.8), which is given for a incidental 1% wage-push in period 1 by:

$$(3.9) \quad \begin{Bmatrix} w \\ r \\ \ell \end{Bmatrix}_t = \left| \frac{\beta_2 B}{A + \beta_2 B} (-\beta_2 \frac{B}{A})^{t-1} + \frac{A}{A + \beta_2 B} \right| \cdot \begin{Bmatrix} 1 \\ -\frac{1}{A} \\ -\frac{B}{A} \end{Bmatrix}$$

The structural value of the model variables is given by:

$$(3.10) \quad \bar{w} = \frac{A}{A + \beta_2 B} P_L$$

$$(3.11) \quad \bar{r} = -\frac{1}{A + \beta_2 B} P_L$$

$$(3.12) \quad \bar{\ell} = -\frac{B}{A + \beta_2 B} P_L$$

The stability and the character of the adjustment process depend on the quantitative values of  $A$ ,  $B$  and  $\beta_2$ . The process is:

- a. oscillatory stable if  $0 < \beta_2 < \frac{A}{B}$
- b. oscillating with constant amplitude if  $\beta_2 = \frac{A}{B}$
- c. unstable with explosive oscillations if  $\beta_2 > \frac{A}{B}$





§4. The process of long term adjustment at full employment level

According to the aggregated demand function (2.11) demand for labour depends - although not exclusively - on the volume of the capital stock that is given and constant on the short run. Via the investment function (2.7) and the accumulation function:

$$(4.1) \quad \tilde{k} = \tilde{k}_{-1}(1-\delta) + \tilde{i}_{-1} \quad ^9)$$

the volume of the capital stock will however change over time. It will be discussed now, whether this volume will reach a constant and stable value in the long run, provided the constancy of labour supply, the preferences and the technical possibilities.

The analysis will be focused on the structural development of the capital stock, so short term adjustment processes as treated in §3 will be ignored. In each period such a distribution of income is supposed to result that full employment of labour is guaranteed. Now the relation between profit-rate and capital stock is easily found:

$$\left. \begin{aligned} (2.11) \quad \tilde{\ell}_v &= \frac{1}{\gamma} \cdot \frac{1}{\kappa_1} (\alpha_1 + D\tilde{r})\tilde{k} \\ (2.9)^a \quad \tilde{\ell}_v &= \frac{\tilde{\ell}}{\tilde{a}} \end{aligned} \right\} \Rightarrow (4.2) \quad \tilde{r} = \frac{\kappa_1 \gamma \tilde{\ell}_a}{D} \quad \frac{1}{\tilde{k}} - \frac{\alpha_1}{D}$$

The net rate of growth of the capital stock is determined by the profit-rate as follows:

$$\left. \begin{aligned} (2.7) \quad \tilde{i} &= \tilde{k} \cdot \tilde{r} \\ (4.1) \quad \tilde{k} &= \tilde{k}_{-1}(1-\delta) + \tilde{i}_{-1} \end{aligned} \right\} \Rightarrow (4.3) \quad g_k = \frac{\tilde{k} - \tilde{k}_{-1}}{\tilde{k}_{-1}} = \tilde{r}_{-1} - \delta$$

<sup>9)</sup>  $\delta$  is the uniform and constant rate of depreciation of the capital stock.

In figure 4 relation (4.2) is drawn. Parallel with the  $\tilde{k}$ -axis the line of the constant rate of depreciation is drawn ( $OA = \delta$ ); so the net rate of growth of the capital stock is found as the difference of both curves.

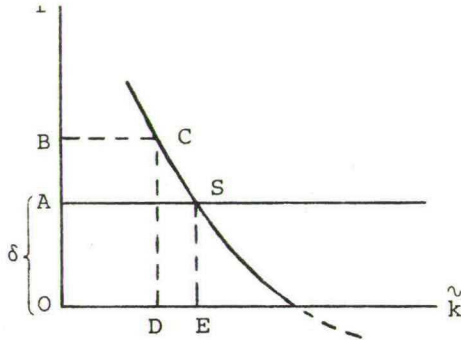


fig. 4. the relation between profit-rate and capital stock

If initially the capital stock is given by  $OD (< OE)$ , a profit-rate  $OB$  is realised higher than the depreciation rate  $OA$ , so the net rate of growth of the capital stock is positive causing a capital stock higher than  $OD$  in the next period. If the initial capital stock were higher than  $OE$ , a negative rate of growth and a smaller capital stock would result.

If  $S$ , which is clearly the structural equilibrium solution of the long term adjustment process and which is characterized by a profit-rate equal to the depreciation rate and a capital stock the size of  $OE$ , is reached in a stable way, is analysed as follows:

From (3.2) and (3.3) the following formula for the adjustment process is derived:

$$(4.4) \quad \tilde{k} = (1 - \delta - \frac{\alpha}{D})k_{-1} + \frac{\kappa_1 \gamma \tilde{L} a}{D}$$

The process is

- a. asymptotically stable if  $0 < \delta + \frac{\alpha}{D} < 1$
- b. quasi stationary if  $\delta + \frac{\alpha}{D} = 1$
- c. oscillatory stable if  $1 < \delta + \frac{\alpha}{D} < 2$
- d. oscillating with constant amplitude if  $\delta + \frac{\alpha}{D} = 2$
- e. unstable with explosive oscillations if  $\delta + \frac{\alpha}{D} > 2$

If equation (4.3) is replaced by the differential notation

$\frac{d\tilde{k}}{\tilde{k}dt} = \tilde{r} - \delta$ , the adjustment process is described by the

following differential equation:

$$(4.5) \quad \frac{d\tilde{k}}{dt} + \left(\frac{\alpha}{D} + \delta\right)\tilde{k} = \frac{\kappa_1 \gamma \tilde{\ell} - a}{D}$$

From the solution

$$(4.6) \quad \tilde{k}_t = \left(\tilde{k}_0 - \frac{\kappa_1 \gamma \tilde{\ell} - a}{\alpha_1 + D}\right) e^{-\left(\frac{\alpha}{D} + \delta\right)t} + \frac{\kappa_1 \gamma \tilde{\ell} - a}{\alpha_1 + D\delta}$$

it follows that the adjustment process is asymptotically stable. If the capital stock has reached the equilibrium value  $OE^{10}$ , the model is that of a stationary economy, (replacement-) investment just compensating the depreciation of the capital stock.

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$$10) \quad OE = \frac{\kappa_1 \gamma \tilde{\ell} - a}{\alpha_1 + D\delta}$$

## §5. The balanced growth model

We ended the preceding paragraph with a stationary economy. Now the characteristics will be studied of an economy which is growing in a balanced way, i.e. that all model variables show a constant - not necessarily the same - rate of growth. Suppose

- a constant periodical (and equal) rise of labour productivity ( $\rho$ ) in sectors  $c_1$  and  $i^{11}$ ). The capital-output ratio is constant.
- a constant periodical decline ( $\eta$ ) of the preference for good  $c_1$ .
- a constant periodical rise ( $\pi$ ) of labour supply.

So we have:

$$(5.1) \quad \frac{1}{\alpha_j}(t) = (1+\rho) \cdot \frac{1}{\alpha_j}(t-1) \quad \text{for } j = i, 1$$

$$(5.2) \quad \gamma(t) = (1-\eta) \cdot \gamma(t-1)$$

$$(5.3) \quad \tilde{\ell}_a(t) = (1+\pi) \cdot \tilde{\ell}_a(t-1).$$

The model of balanced growth - i.e. with full utilisation of both factors of production and a constant profit-rate - is formulated as follows.

From (5.1), (5.2) and the aggregated demand function for labour (2.11) we find:

$$(5.4) \quad g_{\ell_v} \equiv \frac{\tilde{\ell}_v(t) - \tilde{\ell}_v(t-1)}{\tilde{\ell}_v(t-1)} = \frac{1+g_k}{(1+\rho)(1-\eta)} - 1 \approx g_k - \rho + \eta$$

From (5.3) goes:

<sup>11)</sup> Rise of labour productivity in sector  $c_2$  is without consequences for demand for labour; it follows from (2.2)-(2.7) and  $\kappa_2 = 0$  that  $\tilde{\ell}_2 = \alpha_2 \tilde{c}_2 = \frac{1-\gamma}{\gamma} \cdot \frac{1}{\kappa_1} (\alpha_1 + Dr) \cdot \tilde{k}$ .



$$(5.5) \quad g_{\ell_a} = \frac{\tilde{\ell}_a(t) - \tilde{\ell}_a(t-1)}{\tilde{\ell}_a(t-1)} = \pi$$

For a constant profit-rate (3.3) changes to:

$$(5.6) \quad g_k = \tilde{r} - \delta$$

And the condition of full-employment of labour results in the following equilibrium condition:

$$(5.7) \quad g_{\ell_v} = g_{\ell_a}$$

The equations (5.5)-(5.7) describe the balanced growth model. The solution is as follows:

$$(5.8) \quad \tilde{r} = \pi + \rho + \delta - \eta$$

$$(5.9) \quad g_k = \pi + \rho - \eta$$

$$(5.10) \quad g_{\ell_v} = g_{\ell_a} = \pi$$

In table 2 a survey is given of the rates of growth of the model-variables.

Table 2. Survey of the rates of growth in the balanced growth model

rate of growth	variables	rate of growth	variables
0	$\tilde{r}, \tilde{p}_1, \tilde{p}_i$	$\pi + \rho$	$\tilde{\ell}, \tilde{p}_L$
$\pi$	$\tilde{\ell}$	$\pi + \rho + \frac{\gamma_0}{1-\gamma_0} \eta$	$\tilde{c}_2, \tilde{p}_2$
$\rho$	$\tilde{p}_L, \tilde{w}, \tilde{p}_2^{12)}$	$\pi - \eta$	$\tilde{\ell}_1, \tilde{\ell}_i$
$\pi + \rho - \eta$	$\tilde{k}, \tilde{k}p_1, \tilde{r}, \tilde{c}_1, \tilde{c}_1p_1, \tilde{i}, \tilde{i}p_1$	$\pi + \frac{\gamma_0}{1-\gamma_0} \eta$	$\tilde{\ell}_2, \tilde{c}_2^{12)}$

<sup>12)</sup> If there is also a constant rise in labour-productivity ( $\rho$ ) in the  $c_2$ -sector, it follows that:  $g_{p_2} = \rho - \rho_2$  and  $g_{c_2} = \pi + \frac{\gamma_0}{1-\gamma_0} \eta + \rho_2$ .

§6. The interaction between balanced growth and processes of adjustment in the short and in the long run.

The short term dynamics of demand for labour and income distribution for a given capital stock and the long term dynamics of capital stock and income distribution at full-employment level are treated in §3 and §4 respectively. Now a model will be constructed which integrates both types of dynamic adjustment and also takes into account the properties of the balanced growth model of §5.

The model is formulated in terms of relative differences from the equilibrium solution of the balanced growth path. (cfr. §3). So  $\hat{x}$  being the real and  $\hat{x}_E$  being the equilibrium growth value of a variable, the relative difference  $x$  of that variable is defined as:

$$(6.1) \quad x \equiv \frac{\hat{x} - \hat{x}_E}{\hat{x}_E} \quad {}^{13})$$

In the equilibrium neighbourhood the first difference of this new defined variable ( $\Delta x$ )

$$(6.2) \quad \Delta x \equiv x - x_{-1}$$

is approximately equal to the additional or extra rate of growth of the variable with respect to the previous period, that is to say equal to the difference between the actual rate of growth with respect to the previous period and the equilibrium rate of growth of the variable.

Now the factor price frontier (2.10) and the aggregated demand function for labour (2.11) can be reformulated as follows:

$$(6.3) \quad w = -Ar \quad {}^{14})$$

<sup>13)</sup>  $r \equiv \hat{r} - \hat{r}_E$  (cfr. note 2, page 6).

<sup>14)</sup> Second order effects are neglected. The exact solutions are:  $w = -Ar - Bwr$  and  $\ell = k + Br + Bkr$ .

$$(6.4) \quad \ell = k + Br^{14)}$$

It is easy to see that the path of capital accumulation is described as follows:

$$(6.5) \quad \Delta k = r_{-1}$$

As in §3 two versions of labour market reactions can be formulated. In order to distinguish the short term character of labour market reactions from the long term character of capital accumulation, the specification of the labour market reaction curves is without lags:

$$(6.6)^a \quad \Delta w = \beta_1 \ell + \Delta p_L \quad (\text{strong version})$$

$$(6.6)^b \quad \Delta w = \beta_2 \Delta \ell + \Delta p_L \quad (\text{weak version})$$

The model consisting of the equations (6.3), (6.4) and (6.6)<sup>a</sup> can be reduced to a set of difference equations:

$$(6.7) \quad (A + \beta_1 B) \begin{Bmatrix} \ell \\ k \\ w \\ r \end{Bmatrix}_t - (2A + \beta_1 B - \beta_1) \begin{Bmatrix} \ell \\ k \\ w \\ r \end{Bmatrix}_{t-1} + A \begin{Bmatrix} \ell \\ k \\ w \\ r \end{Bmatrix}_{t-2} =$$

$$= \begin{Bmatrix} -B \\ 0 \\ A \\ -1 \end{Bmatrix} \Delta p_{L,t} - \begin{Bmatrix} 1-B \\ 1 \\ A \\ -1 \end{Bmatrix} \Delta p_{L,t-1}$$

For an incidental wage-push the structural or trend solution of the model variables is equal to the equilibrium growth solution. What the stability of the system is concerned, the roots of the characteristic equation have to be studied. This is done in Appendix C. Only for  $B < \frac{1}{2}$ <sup>15)</sup> and relatively high

<sup>14)</sup> See for footnote 14, page 17.

<sup>15)</sup>  $B < \frac{1}{2} \Rightarrow \frac{\alpha_1}{D} + \tilde{r}_0 > 2$ . Cfr. case e, page 14.

values of  $\beta_1$  the system is unstable.

The model consisting of the equations (6.3), (6.4) and (6.6)<sup>b</sup> can also be reduced to a set of difference equations:

$$(6.8) \quad (A + \beta_2 B) \begin{Bmatrix} \ell \\ k \\ w \\ r \end{Bmatrix}_t - (A + \beta_2 B - \beta_2) \begin{Bmatrix} \ell \\ k \\ w \\ r \end{Bmatrix}_{t-1} = \begin{Bmatrix} -B \\ 0 \\ A \\ -1 \end{Bmatrix} p_{Lt} - \begin{Bmatrix} 1-B \\ 1 \\ A \\ -1 \end{Bmatrix} p_{Lt-1}$$

For an incidental wage-push the structural or trend solution is different from the path of balanced growth, at least what demand for labour and capital accumulation is concerned. The income distribution is on the long run not affected by such a wage-push. The structural values of the variables are given by:

$$(6.9) \quad \bar{\ell} = \bar{k} = -\frac{1}{\beta_2} p_L$$

$$(6.10) \quad \bar{w} = \bar{r} = 0.$$

The process is

- a. unstable with explosive oscillations if  $\beta_2 > 2(A + \beta_2 B)$  <sup>16)</sup>
- b. oscillating with constant amplitude if  $\beta_2 = 2(A + \beta_2 B)$
- c. oscillatory stable if  $(A + \beta_2 B) < \beta_2 < 2(A + \beta_2 B)$
- d. quasi stationary if  $\beta_2 = A + \beta_2 B$
- e. asymptotically stable if  $0 < \beta_2 < A + \beta_2 B$

In Appendix D the complete model is presented, i.e. including the equations for the other model variables and including some other disturbances than the wage-push.

The results are illustrated by an example for the weak version

<sup>16)</sup> This is only possible for  $B < \frac{1}{2}$  and  $\beta$  sufficiently high.  
Cfr. Appendix C.

of the labour market reaction curve (6.6)<sup>b</sup>. For the same data as on page 5 and for  $\beta_2 = \frac{3}{2}$  the dynamic behaviour of the real wage-rate is described by:

$$(6.11) \quad w = \frac{7}{8} w_{-1} + \frac{1}{2} \Delta p_L$$

In table 3 the results are summarized of an incidental wage-push of 3% on the various model variables. Because we are dealing with an asymptotically stable process, the presentation of the results for the first two periods and of the structural solution will do.

In table 4 the results are presented in a system of national accounting. For reasons of simplicity the balanced growth model is supposed to generate a stationary economy ( $\pi = 0$ ,  $\rho = 0$ ,  $\eta = 0$ ,  $\delta = 0,1$ ).



Table 3: The results of an incidental wage-push of 3%.

variable ↓ \ period	1	2	trend
k : capital stock	0	$-\frac{1}{4}$	-2
l : demand for labour	-1	$-\frac{9}{8}$	-2
$l_1$ : employment in $c_1$ -sector	$\frac{1}{2}$	$\frac{3}{16}$	-2
$l_2$ : employment in $c_2$ -sector	-1	$-\frac{9}{8}$	-2
$l_i$ : employment in i-sector	$-\frac{5}{2}$	$-\frac{39}{16}$	-2
$c_1$ : volume of $c_1$ -production	$\frac{1}{2}$	$\frac{3}{16}$	-2
$c_2$ : volume of $c_2$ -production	-1	$-\frac{9}{8}$	-2
i : volume of i-production	$-\frac{5}{2}$	$-\frac{39}{16}$	-2
w : real wage rate	$\frac{3}{2}$	$\frac{21}{16}$	0
$p_L$ : nominal wage rate	$\frac{3}{2}$	$\frac{21}{16}$	0
$p_1$ : price of $c_1$ -good	0	0	0
$p_2$ : price of $c_2$ -good	$\frac{3}{2}$	$\frac{21}{16}$	0
$p_i$ : price of i-good	1	$\frac{7}{8}$	0
r : profit-rate	$-\frac{1}{4}$	$-\frac{7}{32}$	0

Table 4: The effects of an incidental wage-push of 3% on the system of national accounting

		equilibrium solution	period 1	trend
wages	$\tilde{l} \cdot \tilde{p}_L$	$40 \times 1 = 40$	$39,6 \times 1,015 = 40,2$	$39,2 \times 1 = 39,2$
profits	$\tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}$	$120 \times 1 \times 0,1 = \underline{12}$ 52	$120 \times 1,01 \times 0,975 = \underline{11,82}$ 52,02	$117,6 \times 1 \times 0,1 = \underline{11,76}$ 50,96
$c_1$ -consumption	$\tilde{c}_1 \cdot \tilde{p}_1$	$20 \times 1 = 20$	$20,1 \times 1 = 20,1$	$19,6 \times 1 = 19,6$
$c_2$ -consumption	$\tilde{c}_2 \cdot \tilde{p}_2$	$20 \times 1 = 20$	$19,8 \times 1,015 = 20,1$	$19,6 \times 1 = 19,6$
investment	$\tilde{i} \cdot \tilde{p}_i$	$12 \times 1 = \underline{12}$ 52	$11,7 \times 1,01 = \underline{11,82}$ 52,02	$11,76 \times 1 = \underline{11,76}$ 50,96
employment $c_1$ -sector	$\alpha_1 \cdot \tilde{c}_1$	$\frac{1}{2} \times 20 = 10$	$\frac{1}{2} \times 20,1 = 10,05$	$\frac{1}{2} \times 19,6 = 9,8$
employment $c_2$ -sector	$\alpha_2 \cdot \tilde{c}_2$	$1 \times 20 = 20$	$1 \times 19,8 = 19,8$	$1 \times 19,6 = 19,6$
employment i-sector	$\alpha_i \cdot \tilde{i}$	$\frac{5}{6} \times 20 = \underline{10}$	$\frac{5}{6} \times 11,7 = \underline{9,75}$	$\frac{5}{6} \times 11,76 = \underline{9,8}$
total employment	$\tilde{l}$	40	39,6	39,2

§7. Some variations on the balanced growth model

An interesting property of the balanced growth model is the rising share of labour-income in total national income for an increasing preference for  $c_2$ -goods, i.e. for  $\eta > 0$  (Cfr. Table 2, page 16). This is due to a switch in the relative shares of the three sectors in the economy, whereas the labour income share per sector is constant.

This is shown in Tables 5 and 6 for a 20-year period for the following data:  $\pi = 0,01$ ;  $\rho = 0,05$ ;  $\delta = 0,05$ ;  $\eta = 0,01$ . For reasons of simplicity the results are presented as if labour-productivity is constant.

Table 5. The balanced growth model (I)

		year 0	year 10	year 20
wages	$\tilde{\tilde{l}}p_L$	$40 \times 1 = 40$	$44 \times 1 = 44$	$48 \times 1 = 48$
profits	$\tilde{\tilde{k}}p_i r$	$120 \times 1 \times 0,1 = \frac{12}{52}$	$120 \times 1 \times 0,1 = \frac{12}{56}$	$120 \times 1 \times 0,1 = \frac{12}{60}$
$c_1$ -consumption	$\tilde{\tilde{c}}_1 \tilde{\tilde{p}}_1$	$20 \times 1 = 20$	$20 \times 1 = 20$	$20 \times 1 = 20$
$c_2$ -consumption	$\tilde{\tilde{c}}_2 \tilde{\tilde{p}}_2$	$20 \times 1 = 20$	$24 \times 1 = 24$	$28 \times 1 = 28$
investment	$\tilde{\tilde{i}} \cdot \tilde{\tilde{p}}_i$	$12 \times 1 = \frac{12}{52}$	$12 \times 1 = \frac{12}{56}$	$12 \times 1 = \frac{12}{60}$
employment				
$c_1$ -sector	$\alpha_1 \tilde{\tilde{c}}_1$	$\frac{1}{2} \times 20 = 10$	$\frac{1}{2} \times 20 = 10$	$\frac{1}{2} \times 20 = 10$
$c_2$ -sector	$\alpha_2 \tilde{\tilde{c}}_2$	$1 \times 20 = 20$	$1 \times 24 = 24$	$1 \times 28 = 28$
i-sector	$\alpha_i \cdot \tilde{\tilde{i}}$	$\frac{5}{6} \times 12 = \frac{10}{6}$	$\frac{5}{6} \times 12 = \frac{10}{6}$	$\frac{5}{6} \times 12 = \frac{10}{6}$
total		40	44	48

Table 6. The balanced growth model (II)

	year	c <sub>1</sub> -sector	c <sub>2</sub> -sector	i-sector	total
share in income (in %)	0	38,5	38,5	23	100
	5	37,0	40,7	22,2	100
	10	35,7	42,9	21,4	100
	15	34,5	44,8	20,7	100
	20	33,3	46,7	20	100
employment (in man-years)	0	10	20	10	40
	5	10	22	10	42
	10	10	24	10	44
	15	10	26	10	46
	20	10	28	10	48
share of labour income (in %)	0	50	100	83,3	76,9
	5	50	100	83,3	77,7
	10	50	100	83,3	78,6
	15	50	100	83,3	79,3
	20	50	100	83,3	80



Now we take into account that the investment function (2.7) can be generalised as follows:

$$(7.1) \quad \tilde{i} = \sigma_R \cdot \tilde{k} \cdot \tilde{r}$$

where  $\sigma_R$  (the propensity to investment with respect to profits) is a function of time, in our case a increasing function.

From (7.1) it follows that

$$(7.2) \quad \frac{\tilde{i}}{\tilde{k}} = \sigma_R \tilde{r}$$

so for a given and constant gross rate of growth of capital this implies a decreasing rate of profit. In this case the rising share of labour-income is not only explained by a switch in the relative shares of the three sectors, but also by a redistribution of income per sector. This is shown in Tables 7 and 8 for a 20-year period for the following data:  $\pi = 0,01$ ;  $\rho = 0,05$ ;  $\delta = 0,05$ ;  $\sigma_R$  is increasing from  $\frac{2}{3}$  to 1 and the decline of  $\gamma$  is just that high that employment in sector  $c_1$  is constant. The gross rate of growth of capital is equal to 10 percent.

To show the influence of the rise of  $\sigma_R$ , the situation in year 20 is given - ceteris paribus - for constant  $\sigma_R$  and linearly declining rate of profit. For reasons of simplicity the price of good  $i$  is set equal to one and the results are presented as if labour-productivity is constant.

Table 7. The balanced growth model with declining rate of profit (I)

		year 0 $\sigma_R = 2/3$	year 10 $\sigma_R = 4/5$	year 20 $\sigma_R = 1$	year 20 *) $\sigma_R = 2/3$
wages	$\tilde{p}_L$	$40 \times 0,9 = 36$	$44 \times 0,95 = 41,8$	$48 \times 1 = 48$	$38,85 \times 1 = 38,85$
profits	$\tilde{p}_i$	$120 \times 1 \times 0,15 = 18$	$120 \times 1 \times 0,125 = 15$	$120 \times 1 \times 0,1 = 12$	$96 \times 1 \times 0,1 = 9,6$
		54	56,8	60	48,45
$c_1$ -consumption	$\tilde{c}_1$	$20 \times 1,2 = 24$	$20 \times 1,1 = 22$	$20 \times 1 = 20$	$17,07 \times 1 = 17,07$
$c_2$ -consumption	$\tilde{c}_2$	$20 \times 0,9 = 18$	$24 \times 0,95 = 22,8$	$28 \times 1 = 28$	$24,99 \times 1 = 24,99$
investment	$\tilde{i} \cdot \tilde{p}_i$	$12 \times 1 = 12$	$12 \times 1 = 12$	$12 \times 1 = 12$	$6,4 \times 1 = 6,4$
		54	56,8	60	48,46
employment					
$c_1$ -sector	$\alpha_1 \tilde{c}_1$	$\frac{1}{2} \times 20 = 10$	$\frac{1}{2} \times 20 = 20$	$\frac{1}{2} \times 20 = 20$	$\frac{1}{2} \times 17,07 = 8,53$
$c_2$ -sector	$\alpha_2 \tilde{c}_2$	$1 \times 20 = 20$	$1 \times 24 = 24$	$1 \times 28 = 28$	$1 \times 24,99 = 24,99$
i-sector	$\alpha_i \tilde{i}$	$\frac{5}{6} \times 12 = 10$	$\frac{5}{6} \times 12 = 10$	$\frac{5}{6} \times 12 = 10$	$\frac{5}{6} \times 6,4 = 5,33$
total		40	44	48	38,85

Table 8. The balanced growth model with declining rate of profit (II)

	year	$\sigma_R$	c <sub>1</sub> -sector	c <sub>2</sub> -sector	i-sector	total
share in income (in %)	0	2/3	44,4	33,3	22,2	100
	5	8/11	41,6	36,8	21,7	100
	10	4/5	38,7	40,1	21,1	100
	15	8/9	36,0	43,4	20,6	100
	20	1	33,3	46,7	20,0	100
	20 *	2/3	35,2	51,6	13,2	100
employment (in man-years)	0	2/3	10	20	10	40
	5	8/11	10	22	10	42
	10	4/5	10	24	10	44
	15	8/9	10	26	10	46
	20	1	10	28	10	48
	20 *	2/3	8,53	24,99	5,33	38,85
share of labour-income (in %)	0	2/3	37,5	100	75,0	66,6
	5	8/11	40,2	100	77,1	70,2
	10	4/5	43,2	100	79,2	73,6
	15	8/9	46,4	100	81,3	76,8
	20	1	50,0	100	83,3	80
	20 *)	2/3	50,0	100	83,3	80,2

As can be read from table 8 the rising share of labour-income in total income is both due to the redistribution of income in sectors  $c_1$  and  $i$  as to a switch in relative position of the sectors.

Because of the condition  $\kappa_2 = 0$  a redistribution of income in sector  $c_2$  is not possible.

If this condition is dropped a redistribution of income in sector  $c_2$  becomes possible. In this case the development of employment and production can only be corresponding to the results of the original model, if the net rate of growth of capital goods is higher than in the original model, because growth of capital is now necessary in the  $c_2$ -sector which is labour-absorbing.

This is illustrated in tables 9 and 10. The fixed coefficients of production are set equal in the sectors  $c_2$  and  $i$ . ( $\alpha_2 = \alpha_1 = 0,8$  and  $\kappa_2 = \kappa_1 = 2$ ; N.B.  $\alpha_1 = 0,6$  and  $\kappa_1 = 4$ ). The gross rate of capital growth is equal to 10 percent and moreover is supposed to be that higher than the sum of rise in labour-productivity - which is the same in all sectors - and rate of depreciation that capital is growing - under the usual presentation as if labour-productivity is constant - by one unit a year. The preference decline for  $c_1$ -goods is just that high that employment in sector  $c_1$  is constant.

Also in this case the consequences of a constant  $\sigma_R$  with declining rate of profit are illustrated.

Table 9. The balanced growth model with declining rate of profit and  $\kappa_2 \neq 0$  (I)

		year 0 $\sigma_R = 2/3$	year 10 $\sigma_R = 4/5$	year 20 $\sigma_R = 1$	year 20 <sup>*)</sup> $\sigma_R = 2/3$
wages	$\tilde{p}_L$	$40 \times 0,875 = 35$	$44 \times 0,9375 = 41,25$	$48 \times 1 = 48$	$32,5 \times 1 = 32,5$
profits	$\tilde{k} \tilde{p}_1 \tilde{r}$	$150 \times 1 \times 0,15 = 22,5$ 57,5	$160 \times 1 \times 0,125 = 20$ 61,25	$170 \times 1 \times 0,1 = 17$ 65	$120 \times 1 \times 0,1 = 12$ 44,5
$c_1$ -consumption	$\tilde{c}_1 \tilde{p}_1$	$20 \times 1,125 = 22,5$	$20 \times 1,0625 = 21,25$	$20 \times 1 = 20$	$15,5 \times 1 = 15,5$
$c_2$ -consumption	$\tilde{c}_2 \tilde{p}_2$	$20 \times 1 = 20$	$24 \times 1 = 24$	$28 \times 1 = 28$	$21 \times 1 = 21$
investment	$\tilde{i} \tilde{p}_i$	$15 \times 1 = 15$ 57,5	$16 \times 1 = 16$ 61,25	$17 \times 1 = 17$ 65	$8 \times 1 = 8$ 44,5
employment					
$c_1$ -sector	$\alpha_1 \tilde{c}_1$	$0,6 \times 20 = 12$	$0,6 \times 20 = 12$	$0,6 \times 20 = 12$	$0,6 \times 15,5 = 9,3$
$c_2$ -sector	$\alpha_2 \tilde{c}_2$	$0,8 \times 20 = 16$	$0,8 \times 24 = 19,2$	$0,8 \times 28 = 22,4$	$0,8 \times 21 = 16,8$
i-sector	$\alpha_i \tilde{i}$	$0,8 \times 15 = 12$	$0,8 \times 16 = 12,8$	$0,8 \times 17 = 13,6$	$0,8 \times 8 = 6,4$
total		40	44	48	32,5



Table 10. The balanced growth model with declining rate of profit and  $\kappa_2 \neq 0$  (II)

	year	$\sigma_R$	$c_1$ -sector	$c_2$ -sector	i-sector	total
share in income (in %)	0	2/3	39,1	34,8	26,1	100
	5	8/11	36,8	37,1	26,1	100
	10	4/5	34,7	39,2	26,1	100
	15	8/9	32,7	41,2	26,1	100
	20	1	30,8	43,1	26,1	100
	20 *	2/3	34,8	47,2	18,0	100
employment (in man-years)	0	2/3	12	16	12	40
	5	8/11	12	17,6	12,4	42
	10	4/5	12	19,2	12,8	44
	15	8/9	12	20,8	13,2	46
	20	1	12	22,4	13,6	48
	20 *	2/3	9,3	16,8	6,4	32,5
share of labour income (in %)	0	2/3	46,7	70	70	60,9
	5	8/11	49,7	72,5	72,5	64,1
	10	4/5	52,9	75	75	67,3
	15	8/9	56,4	77,5	77,5	70,6
	20	1	60	80	80	73,8
	20 *	2/3	60	80	80	73,0

## Appendix A

If the hypothesis of full utilisation of capacity is dropped in the static equilibrium-model and replaced by the hypothesis of exogeneously determined investment, two types of profit-rate will have to be distinguished in the model; a normal or calculation rate of profit ( $\tilde{r}_c$ ), figuring in the price-functions and the factor price frontier derived from it, and a real rate of profit ( $\tilde{r}_f$ ), defined as the ratio between total profits and the value of the stock of capital goods. So total profits can be divided in normal profits, to be calculated as the capital value at normal rate of profit, and surplus profits (or losses), which are the result of over-utilisation (or under-utilisation) of capacity. This is stated in the formulae (a.1) and (a.2). The aggregated demand function for labour and the excess demand function for capacity are given by (a.3) and (a.4).

$$(a.1) \quad \tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_f \equiv \tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_c + \Delta \tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_c$$

$$(a.2) \quad u \equiv 1 + \frac{\Delta \tilde{k}}{\tilde{k}}$$

$$(a.3) \quad \tilde{l}_v = \frac{1}{\kappa_1} \cdot \frac{1}{\gamma} \cdot \frac{\alpha_1 + D\tilde{r}_c}{\tilde{r}_c} \tilde{i}$$

$$(a.4) \quad \Delta \tilde{k} = \frac{\tilde{i}}{\tilde{r}_c} - \tilde{k}$$

$\tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_f$ : total profits

$\tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_c$ : normal profits

$\Delta \tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_c$ : surplus profits

$\tilde{i}$ : exogeneously determined investment

$u$ : rate of utilisation of capacity

$\Delta \tilde{k}$ : excess demand for capacity

If the normal rate of profit and investment are given, the model can be solved. Note that the results are in accordance with Kalecki's theory of profits, which says that 'capitalists earn what they spend, and workers spend what they earn'.\*)

In tables A.1 ( $u > 1$ ), A.2 ( $u = 0$ ) and A.3 ( $u < 1$ ) the solution of the model is registered for the cases of over-employment, full employment and unemployment of labour.

Data:  $\alpha_1 = 0,5$      $\alpha_2 = 1$      $\alpha_i = 0,75$      $\tilde{l}_a = 36$      $\gamma = 0,5$   
          $\kappa_1 = 5$      $\kappa_2 = 0$      $\kappa_i = 2,5$      $\tilde{k} = 105$      $D = 2,5$

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\*) Kaldor, Model of Distribution, in A. Sen, Growth Economics, Penguin Books, 1970.

Table A.1. The model-solution for given normal rate of profit and given investment and more than full utilisation of capacity ( $u > 1$ )

		$\tilde{i} = 13$ $\tilde{r}_C = 0,10$	$\tilde{i} = 12$ $\tilde{r}_C = 0,10$	$\tilde{i} = 11$ $\tilde{r}_C = 0,10$
wages	$\tilde{l}_v \cdot \tilde{p}_L$	$39 \times 1 = 39$	$36 \times 1 = 36$	$33 \times 1 = 33$
normal profits	$\tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_C$	$105 \times 1 \times 0,1 = 10,5$	$105 \times 1 \times 0,1 = 10,5$	$105 \times 1 \times 0,1 = 10,5$
surplus profits	$\Delta \tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_C$	$25 \times 1 \times 0,1 = \frac{2,5}{52}$	$15 \times 1 \times 0,1 = \frac{1,5}{48}$	$5 \times 1 \times 0,1 = \frac{0,5}{44}$
$c_1$ -consumption	$\tilde{c}_1 \cdot \tilde{p}_1$	$19,5 \times 1 = 19,5$	$18 \times 1 = 18$	$16,5 \times 1 = 16,5$
$c_2$ -consumption	$\tilde{c}_2 \cdot \tilde{p}_2$	$19,5 \times 1 = 19,5$	$18 \times 1 = 18$	$16,5 \times 1 = 16,5$
investment	$\tilde{i} \cdot \tilde{p}_i$	$13 \times 1 = \frac{13}{52}$	$12 \times 1 = \frac{12}{48}$	$11 \times 1 = \frac{11}{44}$
employment $c_1$ -sector	$\alpha_1 \cdot \tilde{c}_1$	$0,5 \times 19,5 = 9,75$	$0,5 \times 18 = 9$	$0,5 \times 16,5 = 8,25$
employment $c_2$ -sector	$\alpha_2 \cdot \tilde{c}_2$	$1 \times 19,5 = 19,5$	$1 \times 18 = 18$	$1 \times 16,5 = 16,5$
employment $i$ -sector	$\alpha_i \cdot \tilde{i}$	$0,75 \times 13 = \frac{9,75}{36}$	$0,75 \times 12 = \frac{9}{36}$	$0,75 \times 11 = \frac{8,25}{33}$
total employment	$\tilde{l}_v$	39		
demand for capacity				
in $c_1$ -sector	$\kappa_1 \cdot \tilde{c}_1$	$5 \times 19,5 = 95,5$	$5 \times 18 = 90$	$5 \times 16,5 = 82,5$
in $i$ -sector	$\kappa_i \cdot \tilde{i}$	$2,5 \times 13 = \frac{32,5}{30}$	$2,5 \times 12 = \frac{30}{30}$	$2,5 \times 11 = \frac{27,5}{30}$
total	$\tilde{k} + \Delta \tilde{k}$	130	120	110

Table A.2. The model-solution for given normal rate of profit and given investment and full utilisation of capacity ( $u = 1$ )

		$\tilde{i} = 21$ $\tilde{r}_C = 0,2$	$\tilde{i} = 15$ $\tilde{r}_C = \frac{1}{7}$	$\tilde{i} = 10,5$ $\tilde{r}_C = 0,1$
wages	$\tilde{\ell}_v \cdot \tilde{p}_L$	$42 \times 0,5 = 21$	$36 \times 0,75 = 27$	$31,5 \times 1 = 31,5$
normal profits	$\tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_C$	$105 \times 0,75 \times 0,2 = 15,75$	$105 \times \frac{7}{8} \times \frac{1}{7} = 13,125$	$105 \times 1 \times 0,1 = 10,5$
surplus profits	$\Delta \tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_C$	$\frac{-}{36,75}$	$\frac{-}{40,125}$	$\frac{-}{42}$
$c_1$ -consumption	$\tilde{c}_1 \cdot \tilde{p}_1$	$10,5 \times 1 = 10,5$	$13,5 \times 1 = 13,5$	$15,75 \times 1 = 15,75$
$c_2$ -consumption	$\tilde{c}_2 \cdot \tilde{p}_2$	$21 \times 0,5 = 10,5$	$18 \times 0,75 = 13,5$	$15,75 \times 1 = 15,75$
investment	$\tilde{i} \cdot \tilde{p}_i$	$21 \times 0,75 = \frac{15,75}{36,75}$	$15 \times \frac{7}{8} = \frac{13,125}{40,125}$	$10,5 \times 1 = \frac{10,5}{42}$
employment $c_1$ -sector	$\alpha_1 \cdot \tilde{c}_1$	$0,5 \times 10,5 = 5,25$	$0,5 \times 13,5 = 6,75$	$0,5 \times 15,75 = 7,875$
employment $c_2$ -sector	$\alpha_2 \cdot \tilde{c}_2$	$1 \times 21 = 21$	$1 \times 18 = 18$	$1 \times 15,75 = 15,75$
employment i-sector	$\alpha_i \cdot \tilde{i}$	$0,75 \times 21 = \frac{15,75}{36,75}$	$0,75 \times 15 = \frac{11,25}{40,125}$	$0,75 \times 10,5 = \frac{7,875}{42}$
total employment	$\tilde{\ell}_v$	42	36	31,5
demand for capacity				
in $c_1$ -sector	$\kappa_1 \cdot \tilde{c}_1$	$5 \times 10,5 = 52,5$	$5 \times 13,5 = 67,5$	$5 \times 15,75 = 78,75$
in i-sector	$\kappa_i \cdot \tilde{i}$	$2,5 \times 21 = \frac{52,5}{36,75}$	$2,5 \times 15 = \frac{37,5}{40,125}$	$2,5 \times 10,5 = \frac{26,25}{42}$
total	$\tilde{k} + \Delta \tilde{k}$	105	105	105



Table A.3. The model-solution for given normal rate of profit and given investment and less than full utilisation of capacity ( $u < 1$ )

		$\tilde{i} = 20$ $\tilde{r}_C = 0,2$	$\tilde{i} = 18$ $\tilde{r}_C = 0,2$	$\tilde{i} = 16$ $\tilde{r}_C = 0,2$
wages	$\tilde{l}_v \cdot \tilde{p}_L$	$40 \times 0,5 = 20$	$36 \times 0,5 = 18$	$32 \times 0,5 = 16$
normal profits	$\tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_C$	$105 \times 0,75 \times 0,2 = 15,75$	$105 \times 0,75 \times 0,2 = 15,75$	$105 \times 0,75 \times 0,2 = 15,75$
surplus profits	$\Delta \tilde{k} \cdot \tilde{p}_i \cdot \tilde{r}_C$	$\div 5 \times 0,75 \times 0,2 = \frac{\div 0,75}{35}$	$\div 15 \times 0,75 \times 0,2 = \frac{\div 2,25}{31,5}$	$\div 25 \times 0,75 \times 0,2 = \frac{\div 3,75}{28}$
$c_1$ -consumption	$\tilde{c}_1 \cdot \tilde{p}_1$	$10 \times 1 = 10$	$9 \times 1 = 9$	$8 \times 1 = 8$
$c_2$ -consumption	$\tilde{c}_2 \cdot \tilde{p}_2$	$20 \times 0,5 = 10$	$18 \times 0,5 = 9$	$16 \times 0,5 = 8$
investment	$\tilde{i} \cdot \tilde{p}_i$	$20 \times 0,75 = \frac{15}{35}$	$18 \times 0,75 = \frac{13,5}{31,5}$	$16 \times 0,75 = \frac{12}{28}$
employment $c_1$ -sector	$\alpha_1 \cdot \tilde{c}_1$	$0,5 \times 10 = 5$	$0,5 \times 9 = 4,5$	$0,5 \times 8 = 4$
employment $c_2$ -sector	$\alpha_2 \cdot \tilde{c}_2$	$1 \times 20 = 20$	$1 \times 18 = 18$	$1 \times 16 = 16$
employment i-sector	$\alpha_i \cdot \tilde{i}$	$0,75 \times 20 = \frac{15}{35}$	$0,75 \times 18 = \frac{13,5}{31,5}$	$0,75 \times 16 = \frac{12}{28}$
total employment	$\tilde{l}_v$	40	36	32
demand for capacity				
in $c_1$ -sector	$\kappa_1 \cdot \tilde{c}_1$	$5 \times 10 = 50$	$5 \times 9 = 45$	$5 \times 8 = 40$
in i-sector	$\kappa_i \cdot \tilde{i}$	$2,5 \times 20 = \frac{50}{35}$	$2,5 \times 18 = \frac{45}{31,5}$	$2,5 \times 16 = \frac{40}{28}$
total	$\tilde{k} + \Delta \tilde{k}$	100	90	80

## Appendix B

In this appendix the algebraic formulation of the curves (a)-(a), (figure 2, page 9 and figure 3, page 11) is derived. An alternative graphic representation of the short term adjustment process is also developed.

The curve (a)-(a) for the strong version of labour-market reactions is based on the relations (2.10), (2.11) and (3.5)<sup>a</sup> and on the definitions (3.1) and (3.2).

$$\left. \begin{aligned} (2.10) \quad \tilde{w} &= \frac{1 - \kappa_1 \tilde{r}}{\alpha_1 + D\tilde{r}} \\ (2.11) \quad \ell_v &= \frac{1}{\gamma \kappa_1} (\alpha_1 + D\tilde{r}) \tilde{k} \Rightarrow \tilde{r} = \frac{\gamma \kappa_1}{D \tilde{k}} \tilde{\ell}_v + \frac{\alpha_1}{D} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow (I) \quad \tilde{w} = \frac{\kappa_1 \alpha_1 \tilde{k}}{\gamma \kappa_1 D} \cdot \frac{1}{\tilde{\ell}_v} - \frac{\kappa_1}{D}$$

$$(3.5)^a \quad \Delta w = \beta_1 \ell_{-1} + \Delta p_L$$

$$\frac{\tilde{w} - \tilde{w}_0}{\tilde{w}_0} - \frac{\tilde{w}_{-1} - \tilde{w}_0}{\tilde{w}_0} = \beta_1 \frac{\tilde{\ell}_{v-1} - \tilde{\ell}_0}{\tilde{\ell}_0} + \frac{\tilde{p}_L - \tilde{p}_{L0}}{\tilde{p}_{L0}} - \frac{\tilde{p}_{L-1} - \tilde{p}_{L0}}{\tilde{p}_{L0}}$$

$$\tilde{w} = \tilde{w}_{-1} + \beta_1 \frac{\tilde{w}_0}{\tilde{\ell}_0} \tilde{\ell}_{v-1} - \beta_1 \tilde{w}_0 + (\tilde{p}_L - \tilde{p}_{L-1})$$

Taking into account (I) the curve (a)-(a) is found:<sup>1)</sup>

$$\tilde{w}_{+1} = \frac{\kappa_1 \alpha_1 \tilde{k}}{\gamma \kappa_1 D} \cdot \frac{1}{\tilde{\ell}_v} + \beta_1 \frac{\tilde{w}_0}{\tilde{\ell}_0} \tilde{\ell}_v - \frac{\kappa_1}{D} - \beta_1 \tilde{w}_0 + (\tilde{p}_{L+1} - \tilde{p}_L)$$

<sup>1)</sup> All variables are shifted one period.

This function, being a combination of a hyperbole and a straight line, will reach an extreme solution if the first derivative vanishes.

$$\frac{d\tilde{w}_{+1}}{d\tilde{\ell}_v} = - \frac{\kappa_1 \alpha_1 \tilde{k}}{\gamma \kappa_1 D} \cdot \frac{1}{\tilde{\ell}_v^2} + \frac{\beta_1 \tilde{w}_0}{\tilde{\ell}_0}$$

By substitution of (2.10), (2.11) and the definitions of A and B - see (3.3) and (3.4), page 7 - we get:

$$\begin{aligned} \frac{d\tilde{w}_{+1}}{d\tilde{\ell}_v} &> 0 \quad \text{if} \quad \tilde{\ell}_v > \tilde{\ell}_0 \sqrt{\frac{A}{B\beta_1}} \quad \text{or} \quad \tilde{\ell}_v < -\tilde{\ell}_0 \sqrt{\frac{A}{B\beta_1}} \\ &= 0 \quad \text{if} \quad \tilde{\ell}_v = \pm \tilde{\ell}_0 \sqrt{\frac{A}{B\beta_1}} \\ &> 0 \quad \text{if} \quad -\tilde{\ell}_0 \sqrt{\frac{A}{B\beta_1}} < \tilde{\ell}_v < \tilde{\ell}_0 \sqrt{\frac{A}{B\beta_1}} \end{aligned}$$

The curve (a)-(a) is drawn below for these three situations<sup>2)</sup>

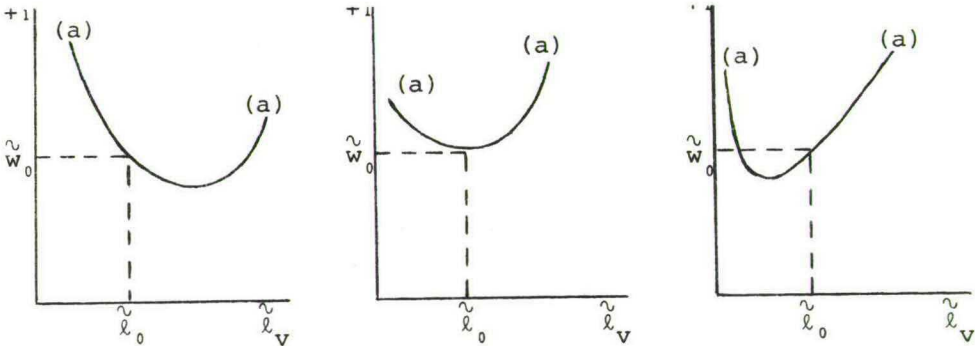


fig. a: curve (a)·(a) for  $\beta_1 < \frac{A}{B}$       fig. b: curve (a)·(a) for  $\beta_1 = \frac{A}{B}$       fig. c: curve (a)·(a) for  $\beta_1 > \frac{A}{B}$

<sup>2)</sup> The negative function-area has been omitted for it is economically not relevant.

In these diagrams<sup>3)</sup> curve I, representing the relation between demand for labour and the real wage-rate, is added.

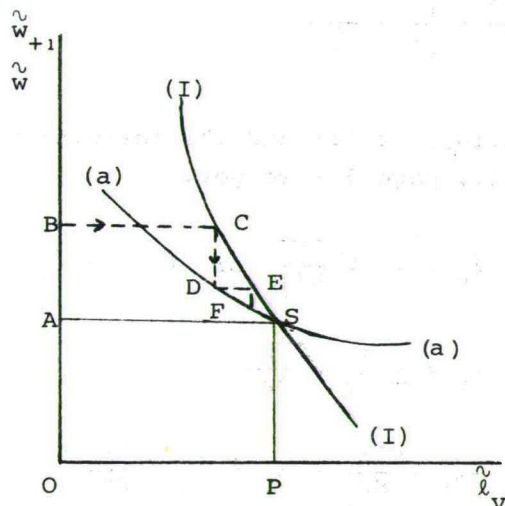


fig a': curves (a) and (I)

$$\text{for } \beta_1 < \frac{A}{B}$$

<sup>3)</sup> As stated in note 3, page 7 a second order effect in formule (3.3):  $w = -Ar$  has been neglected; the exact solution ( $w = -Ar - Bwr$ ) would provide a non-linear difference equation:  $\Delta w = -\beta_1 \frac{Bw_{-1}}{A+Bw_{-1}} + \Delta p_L$ . The stability conditions and the character of the adjustment process that is denoted by this formula are slightly different from the descriptions and conditions on page 8. Now not only the quantitative values of A, B and  $\beta_1$ , but also the size of the initial disturbance  $\Delta p_L$  determinate the characteristics of the adjustment process. For instance the process is quasi stationary if  $\beta_1 = \frac{A}{B} + \Delta p_L$ . These complications present difficulties especially in the figure b situation, so in order to avoid interrupting the story unnecessarily with technical details, figure b is ignored from now on.

In figure a' the asymptotically stable process (case a, page 8) can be seen. A disturbance of the equilibrium wage-rate ( $OB > OA$ ) reduces demand for labour ( $BC < OP$ ); reactions in the labour-market via (a)-(a) push down the high wage-rate  $OB$  by  $CD$ , but not enough to reach equilibrium. Demand for labour is increasing (by  $DE$ ) but full employment is not reached. This process of falling wages and increasing employment goes on until (asymptotically) the equilibrium wage-rate ( $OA$ ) and full employment ( $OP$ ) are realised.

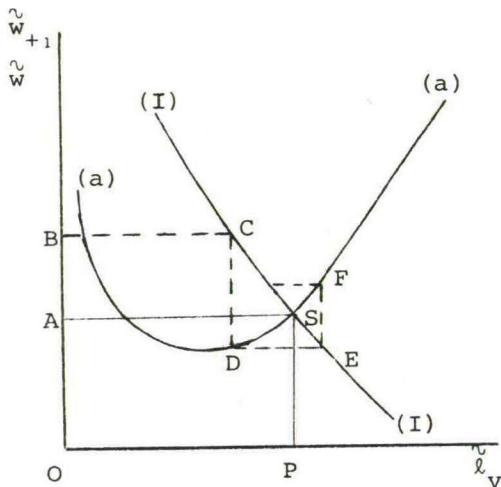


fig c': curves (a) • (a) and (I)  
for  $\beta_1 > \frac{A}{B}$

In figure c' you can see the oscillatory stable process (case c, page 8).

In this situation too a disturbance of the equilibrium wage-rate ( $OB > OA$ ) initially reduces demand for labour ( $BC < OP$ ); reactions in the labour-market via (a)-(a) are so vigorous that the high wage-rate  $OB$  is pushed down by  $CD$  under the equilibrium wage-rate. Demand for labour is increasing above the full employment level, so in the next round wages are

pushed up again. If the right part of curve (a)-(a) is not too steep - i.e. for  $\beta_1$  not valued too high - the result is a stable cobweb towards the equilibrium S. If  $\beta_1$  is valued high the process of adjustment becomes unstable with explosive oscillations. The oscillating process with constant amplitude is in between the former and the latter situation.

For the weak version of labour-market-reactions curve (a)-(a) is based on relations (2.10), (2.11) and (3.5)<sup>b</sup> and on definitions (3.1) and (3.2).

$$\tilde{w}_{+1} = \beta_2 \frac{\tilde{w}_0}{\tilde{l}_0} \tilde{l}_v - \beta_2 \tilde{w}_0 + \tilde{p}_{L+1}$$

In figure d this curve (a)-(a) is drawn together with curve I, representing the relation between demand for labour and the real wage-rate .

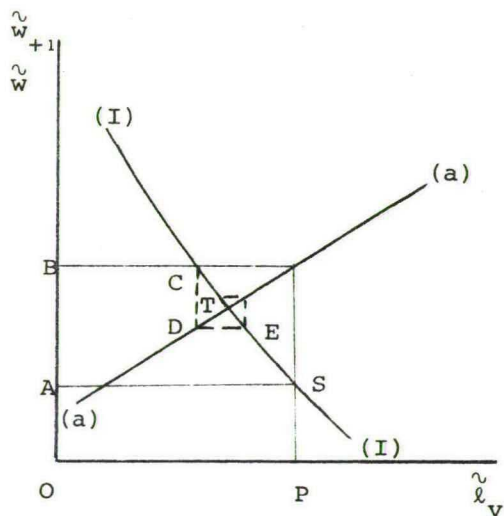


fig d.: curves (a)-(a) and (I)  
for the weak version

The character of the oscillatory stable adjustment process (case a, page 10) can easily be read from figure d. A disturbance of the equilibrium wage-rate ( $\tilde{w}_0 = OA$  and  $\tilde{p}_L = OB$ ) reduces demand for labour ( $BC < OP$ ), Reactions in the labour-



market via (a)-(a) push down the high wage-rate BC by CD, but the equilibrium wage-rate (OA) is not reached. Demand for labour is increasing (by DE), but not enough to realise full employment. This process goes on in a stable cobweb-way until the stability point T that is characterized by unemployment and a wage-rate higher than equilibrium-rate OA, but lower than the level OB caused by initial disturbance, is reached. If  $\beta_2$  is valued higher, i.e. curve (a)-(a) proceeds steeper, the process of adjustment is oscillatory with constant amplitude or unstable with explosive oscillations.

Appendix C: The stability of difference equations (6.7)

Consider:

$$(c.1) \quad (A+\beta B)X_t - (2A+\beta B-\beta)X_{t-1} + AX_{t-2} = f(t);$$

$$A > 0, B > 0, \beta > 0 \quad [\text{Cfr (6.7)}]$$

The characteristic equation of this difference equation is as follows

$$(c.2) \quad (A+\beta B)\lambda^2 - (2A+\beta B-\beta)\lambda + A = 0.$$

The discriminant (D) of this quadratic equation is as follows:

$$(c.3) \quad D = \{2A+\beta B-\beta\}^2 - 4A(A+\beta B)$$

$$= \beta^2 (B-1)^2 - 4A\beta$$

$$= \beta\{\beta(B-1)^2 - 4A\}$$

$$\text{Now } \beta > 0, \text{ hence } D < 0 \text{ if } 0 < \beta < \frac{4A}{(B-1)^2}$$

$$D = 0 \text{ if } \beta = \frac{4A}{(B-1)^2}$$

$$D > 0 \text{ if } \beta > \frac{4A}{(B-1)^2}$$

I If  $D < 0$  the square roots of the characteristic equation (c.2) are complex numbers, so the process is surely stable if  $\frac{A}{A+\beta B} < 1$ . For  $A > 0, B > 0$  and  $\beta > 0$  this is true.

II If  $D = 0$  the square roots of the characteristic equation (c.2) are the same and equal to:

$$\left. \begin{aligned} \lambda_{1,2} &= \frac{2A + \beta(B-1)}{2A + 2\beta B} \\ \text{but } \beta &= \frac{4A}{(B-1)^2} \end{aligned} \right\} \Rightarrow \lambda_{1,2} = \frac{B-1}{B+1}$$

For  $B > 0$  it follows  $-1 < \lambda_{1,2} < 1$ , so the process is stable.

III If  $D > 0$  the square roots of the characteristic equation (c.2) are different and equal to:

$$\lambda_{1,2} = \frac{2A + \beta(B-1) \pm \sqrt{\beta^2(B-1)^2 - 4A\beta}}{2A + 2\beta B}$$

Now the process is stable if a)  $\lambda_1 < 1$  and b)  $\lambda_2 > -1$   
( $\lambda_1 > \lambda_2$ )

$$\begin{aligned} \text{a)} \quad & \lambda_1 < 1 \\ & \frac{2A + \beta(B-1) + \sqrt{\beta^2(B-1)^2 - 4A\beta}}{2A + 2\beta B} < 1 \end{aligned}$$

$$2A + \beta B - \beta + \sqrt{\beta^2(B-1)^2 - 4A\beta} < 2A + 2\beta B$$

$$\sqrt{\beta^2(B-1)^2 - 4A\beta} < \beta(B+1)$$

$$\beta^2(B-1)^2 - 4A\beta < \beta^2(B+1)^2$$

$$\beta^2(-4B) < 4A\beta$$

$$\beta > -\frac{A}{B}$$

but  $A > 0$ ,  $B > 0$  and  $\beta > 0$ , so  $\lambda_1 < 1$  is always true.

b)

$$\lambda_2 > -1$$

$$\frac{2A + \beta(B-1) - \sqrt{\beta^2(B-1)^2 - 4A\beta}}{2A + 2\beta B} > -1$$

$$2A + \beta B - \beta - \sqrt{\beta^2(B-1)^2 - 4A\beta} > -2A - 2\beta B$$

$$\sqrt{\beta^2(B-1)^2 - 4A\beta} < 4A + \beta(3B-1)$$

If  $4A + \beta(3B-1) \leq 0$  - which is only possible for  $0 < B < \frac{1}{3}$  and  $\beta$  sufficiently high - then  $\lambda_2 < -1$  and the process is unstable.

If  $4A + \beta(3B-1) > 0$ , both sides of the inequality are squared, so

$$\beta^2(B-1)^2 - 4A\beta < \{4A + \beta(3B-1)\}^2$$

$$\beta^2(B-2B^2) + \beta(A-6AB) - 4A^2 < 0.$$

Consider function  $f(\beta) = \beta^2(B-2B^2) + \beta(A-6AB) - 4A^2$

$$f(\beta) = 0 \quad \text{for} \quad \beta_1 = \frac{4A}{1-2B} \quad \text{and} \quad \beta_2 = -\frac{A}{B} < 0$$

$$f(0) = -4A^2 < 0.$$

$\alpha)$  If  $B > \frac{1}{2}$ , then  $1-2B < 0$ , so  $\beta_1 < \beta_2 < 0$ .

, then  $B-2B^2 < 0$ , so  $f(\beta)$  is a parabola with downwards pointing axis.

(fig. 1)

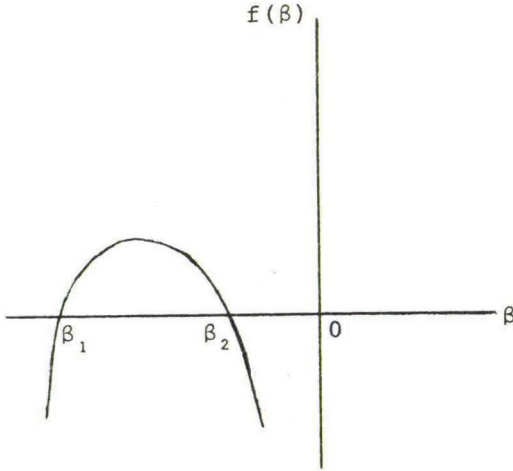


fig. 1.

So  $f(\beta) < 0$  for all  $\beta > 0$ , hence  $\lambda_2 > -1$  and the process is stable.

$\beta$ ) If  $B = \frac{1}{2}$ , then  $f(\beta) = -2A\beta - 4A^2 < 0$  for all  $\beta > 0$ , so  $\lambda_2 > -1$  and the process is stable.

$\gamma$ ) If  $0 < B < \frac{1}{2}$ , then  $1 - 2B > 0$ , so  $\beta_1 > 0 > \beta_2$ ,  
 , then  $B - 2B^2 > 0$ , so  $f(\beta)$  is a parabola with  
 upwards pointing axis.  
 (fig. 2)

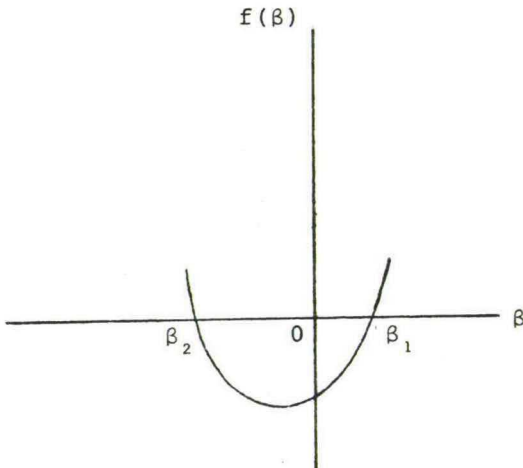


fig. 2.

So  $f(\beta) < 0$  (hence  $\lambda_2 > -1$ ), if  $0 < \beta < \beta_1$

hence 
$$\beta < \frac{4A}{1-2B}$$

$$4A > \beta(1-2B)$$

$$4A + \beta(2B-1) > 0$$

[this result implies  
condition  $4A + \beta(3B-1) > 0$ ]

so if  $0 < B \leq \frac{1}{3} \Rightarrow \beta < \frac{4A}{1-2B}$

and if  $\frac{1}{3} < B < \frac{1}{2} \Rightarrow \beta < \frac{4A}{1-2B}$

The general conclusion is that the process is stable for  $\beta \geq \frac{1}{2}$  and that for  $0 < B < \frac{1}{2}$  instability is only possible for relatively high values of  $\beta$ .



# Appendix D.

The complete model is as follows<sup>1)</sup>

$$(1) \quad r = -\frac{1}{A} (w - \underline{\rho}) \quad A = \frac{\kappa_1 \alpha_i}{(\alpha_1 + D\tilde{r}_0)(1 - \kappa_i \tilde{r}_0)} > 0$$

$$(2) \quad \ell = k + Br + \underline{\eta} - \underline{\rho} - \underline{\pi} \quad B = \frac{D}{\alpha_1 + D\tilde{r}_0} > 0$$

$$(3) \quad \Delta k = r_{-1}$$

$$(4)^a \quad \Delta w = \beta_1 \ell + \Delta p_L$$

$$(4)^b \quad \Delta w = \beta_2 \Delta \ell + \Delta p_L$$

$$(5) \quad p_1 = 0$$

$$(6) \quad p_L = w + p_1$$

$$(7) \quad p_2 = p_L$$

$$(8) \quad p_i = p_L + \frac{\kappa_i}{1 - \kappa_i \tilde{r}_0} r - \underline{\rho}$$

$$(9) \quad c_1 = \ell + p_L - p_1 + \underline{\pi} - \underline{\eta}$$

<sup>1)</sup> The variable  $\ell$  should be interpreted as the relative difference from a situation of full employment; the other variables are as usual, i.e. relative differences from the balanced growth path.

$$(10) \quad c_2 = \ell + p_L - p_2 + \pi + \frac{\gamma_0}{1-\gamma_0} \eta$$

$$(11) \quad i = k + \frac{r}{r_0}$$

$$(12) \quad \ell_1 = c_1 - \rho - \pi$$

$$(13) \quad \ell_2 = c_2 - \pi$$

$$(14) \quad \ell_i = i - \rho - \pi$$

The disturbances are defined as follows

1a. incidental wage push	$\Delta p_{Lt} \neq 0$ for $t = 1$ $= 0$ for $t \neq 1$
1b. permanent wage push	$\Delta p_{Lt} \neq 0$ for all $t$
2a. incidental extra rise in productivity	$\Delta p_t \neq 0$ for $t = 1$ $= 0$ for $t \neq 1$
2b. permanent extra rise in productivity	$\Delta p_t \neq 0$ for all $t$
3a. incidental extra rise in labour supply	$\Delta \pi_t \neq 0$ for $t = 1$ $= 0$ for $t \neq 1$
3b. permanent extra rise in labour supply	$\Delta \pi_t \neq 0$ for all $t$
4a. incidental extra preference shift	$\Delta \eta_t \neq 0$ for $t = 1$ $= 0$ for $t \neq 1$
4b. permanent extra preference shift	$\Delta \eta_t \neq 0$ for all $t$ .

For (4)<sup>a</sup> the difference equation in  $\ell$  is as follows:

$$\begin{aligned}
 (A+\beta_1 B)\ell - (2A+\beta_1 B-\beta_1)\ell_{-1} + A\ell_{-2} &= \\
 = -B(\Delta p_L - \Delta \rho) - (1-B)(\Delta p_{L_{-1}} - \Delta \rho_{-1}) + A(\Delta \eta - \Delta \rho - \Delta \pi) - A(\Delta \eta_{-1} - \Delta \rho_{-1} - \Delta \pi_{-1})
 \end{aligned}$$

For (4)<sup>b</sup> the difference equation in  $\ell$  is as follows:

$$\begin{aligned}
 (A+\beta_2 B)\ell - (A+\beta_2 B-\beta_2)\ell_{-1} &= \\
 = -B(p_L - \rho) - (1-B)(p_{L_{-1}} - \rho_{-1}) + A(\Delta \eta - \Delta \rho - \Delta \pi)
 \end{aligned}$$

The structural results of the four types of disturbances are summarized in the table for the weak version (4)<sup>b</sup> case. The value of the various impulses is always 1,5%.

variable + / disturbance	1a	1b	2a	2b	3a	3b	4a	4b
k : capital stock	-1	$\Delta-1$	2,5	$\Delta 2,5$	1	$\Delta 1$	-1	$\Delta-1$
$\ell$ : demand for labour	-1	$\Delta-1$	1	$\Delta 1$	0	-6	0	6
$\ell_1$ : employment in $c_1$ -sector	-1	$\Delta-1$	1	$\Delta 1$	0	-15	-1	$\Delta-1$
$\ell_2$ : employment in $c_2$ -sector	-1	$\Delta-1$	1	$\Delta 1$	0	-6	1	$\Delta+1$
$\ell_i$ : employment in i-sector	-1	$\Delta-1$	1	$\Delta 1$	0	3	-1	$\Delta-1$
$c_1$ : volume of $c_1$ -production	-1	$\Delta-1$	2,5	$\Delta 2,5$	1	$\Delta 1$	-1	$\Delta-1$
$c_2$ : volume of $c_2$ -production	-1	$\Delta-1$	1	$\Delta 1$	1	$\Delta 1$	-1	$\Delta-1$
i : volume of i-production	-1	$\Delta-1$	2,5	$\Delta 2,5$	1	$\Delta 1$	-1	$\Delta-1$
w : real wage-rate	0	6	1,5	$\Delta 1,5$	0	-9	0	9
$p_L$ : nominal wage-rate	0	6	1,5	$\Delta 1,5$	0	-9	0	9
$p_1$ : price of $c_1$ -good	0	0	0	0	0	0	0	0
$p_2$ : price of $c_2$ -good	0	6	1,5	$\Delta 1,5$	0	-9	0	9
$p_i$ : price of i-good	0	4	0	-6,67	0	-6	0	6
r : profit-rate	0	-1	0	2,5	0	1	0	-1







## PREVIOUS NUMBERS:

- |   |  |
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\*) not available